On quadratic and almost quadratic p-adic and adelic path integrals in adelic quantum mechanics

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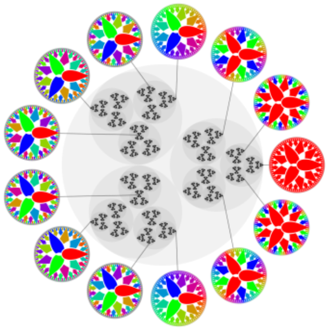


Goran S. Djordjević

In cooperation with

D. Delibašić, D. Dimitrijević and M. Milošević

SEENET-MTP Centre & Department of Physics,
Faculty of Science and Mathematics
University of Niš, Serbia

p-Adic and Adelic Path Integrals
Quantum Mechanics and Cosmological Inflation

- Introduction and motivation
- *p*-Adic and Adelic Quantum Mechanics
- *p*-Adic and Adelic Path Integrals – Quadratic Systems
- Tachyons and Inflation
- Tachyons-From Field Theory to Classical Analogue – DBI and Sen approach
- Tachyon–like inflation in a Rundall-Sundrum approach
- Beyond?

Introduction and Motivation

- The main task of quantum cosmology is to describe the evolution of the universe in a very early stage.
- Since quantum cosmology is related to the Planck scale phenomena it is logical to consider various geometries (in particular nonarchimedean, noncommutative …)
- Despite some evident problems such as a non-sufficiently long period of inflation, tachyon-driven scenarios, both real and *p*-adic, remain highly interesting for study.

Adelic Quantum Theory

- Reasons to use *p*-adic numbers and adeles in quantum physics:
- The field of rational numbers **Q**, which contains all observational and experimental numerical data, is a dense subfield not only in **R** but also in the fields of *p*-adic numbers Q_p.
- There is an analysis within and over $\boldsymbol{Q_p}$ like that one related to **R.**
- General mathematical methods and fundamental physical laws should be invariant [I.V. Volovich, (1987), Vladimirov, Volovich, (1994)] under an interchange of the number fields **R** and $\boldsymbol{Q_p}$.
- There is a quantum gravity uncertainty ($\Delta x \geq l_0 = \sqrt{\hbar G / c^3}$), when measures distances around the Planck length , which restricts priority of Archimedean geometry based on the real numbers and gives rise to employment of non-Archimedean geometry.
- It seems to be quite reasonable to extend standard Feynman's path integral method to non-Archimedean spaces.

Adelic Quantum Theory –
p-ADIC FUNCTIONS AND INTEGRATION

- There are primary two kinds of analyses on **Q***p* *:* **Q***p* → **Q***p* (class.) and **Q***p* → **C** (quant.).
- Usual complex valued functions of p-adic variable, which are employed in mathematical physics, are :
- an additive character $\chi_p(x) = \exp 2\pi i \{x\}_p,$
- locally constant functions with compact support

$$\Omega(|x|_p) = \begin{cases} 1 & |x|_p \leq 1 \\ 0 & |x|_p > 1 \end{cases}$$

Adelic Quantum Theory –

p-ADIC FUNCTIONS AND INTEGRATION

- There is well defined Haar measure and integration. Important integrals are

$$\int_{Q_p} \chi_p(ayx)dx = \delta_p(ay) = |a|_p^{-1} \delta_p(y), \; a \neq 0$$

$$\int_{Q_p} \chi_p(\alpha x^2 + \beta x)dx = \lambda_p(\alpha) |2\alpha|_p^{-1/2} \chi_p\left(-\frac{\beta^2}{4\alpha}\right), \; \alpha \neq 0$$

- Real analogues of integrals

$$\int_{Q_\infty} \chi_\infty(ayx)dx = \delta_\infty(ay) = |a|_\infty^{-1} \delta_\infty(y), \; a \neq 0$$

$$\int_{Q_\infty} \chi_\infty(\alpha x^2 + \beta x)dx = \lambda_\infty(\alpha) |2\alpha|_\infty^{-1/2} \chi_\infty\left(-\frac{\beta^2}{4\alpha}\right), \; \alpha \neq 0$$

$$Q_\infty \equiv R, \quad \chi_\infty(x) = \exp(-2\pi i x)$$

Adelic Quantum Theory –
Dynamics of a *p*-adic quantum model

- **Dynamics of *p*-adic quantum model**
- *p*-adic quantum mechanics is given by a triple

$$(L_2(Q_p), W_p(z_p), U_p(t_p))$$

- Adelic evolution operator is defined by

$$U(t)\psi(x) = \int_A K_t(x,y)\psi(y)dy = \prod_{v=\infty,2,3,\ldots,p,\ldots} \int_{Q_v} K_t^v(x_v, y_v)\psi^{(v)}(y_v)dy_v$$

- The eigenvalue problem

$$U(t)\psi_\alpha(x) = \chi(E_\alpha t)\psi_\alpha(x)$$

Adelic Quantum Theory – path integrals

- The main problem in our approach is computation of *p*-adic transition amplitude in Feynman's PI method

$$K_p(x'',t'';x',t') = \int_{(x',t')}^{(x'',t'')} \chi_p\left(-\frac{1}{h}\int_{t'}^{t''} L(\dot{q},q,t)\right) Dq$$

- Exact general expression ($\bar{S}$-classical action)

$$K_p(x'',t'';x',t') = \lambda_p\left(-\frac{1}{2h}\frac{\partial^2 \bar{S}}{\partial x'\partial x''}\right) \times \left|\frac{1}{h}\frac{\partial^2 \bar{S}}{\partial x'\partial x''}\right|_p^{1/2} \chi_p\left(-\bar{S}(x'',t'';x',t')\right)$$

$$K_p(x'',y'',z'',t'';x',y',z',t') = \lambda_p\left(\det\begin{pmatrix} -\frac{1}{2}\frac{\partial^2 \bar{S}}{\partial x'\partial x''} & -\frac{1}{2}\frac{\partial^2 \bar{S}}{\partial x'\partial y''} & -\frac{1}{2}\frac{\partial^2 \bar{S}}{\partial x'\partial z''} \\ -\frac{1}{2}\frac{\partial^2 \bar{S}}{\partial y'\partial x''} & -\frac{1}{2}\frac{\partial^2 \bar{S}}{\partial y'\partial y''} & -\frac{1}{2}\frac{\partial^2 \bar{S}}{\partial y'\partial z''} \\ -\frac{1}{2}\frac{\partial^2 \bar{S}}{\partial z'\partial x''} & -\frac{1}{2}\frac{\partial^2 \bar{S}}{\partial z'\partial y''} & -\frac{1}{2}\frac{\partial^2 \bar{S}}{\partial z'\partial z''} \end{pmatrix}\right)$$

$$\times \left|\det\begin{pmatrix} -\frac{\partial^2 \bar{S}}{\partial x'\partial x''} & -\frac{\partial^2 \bar{S}}{\partial x'\partial y''} & -\frac{\partial^2 \bar{S}}{\partial x'\partial z''} \\ -\frac{\partial^2 \bar{S}}{\partial y'\partial x''} & -\frac{\partial^2 \bar{S}}{\partial y'\partial y''} & -\frac{\partial^2 \bar{S}}{\partial y'\partial z''} \\ -\frac{\partial^2 \bar{S}}{\partial z'\partial x''} & -\frac{\partial^2 \bar{S}}{\partial z'\partial y''} & -\frac{\partial^2 \bar{S}}{\partial z'\partial z''} \end{pmatrix}\right|_p^{1/2} \chi_p\left(-\bar{S}(x'',y'',z'',t'';x',y',z't')\right)$$

Adelic Quantum Theory

- Adelic quantum mechanics [Dragovich (1994), G. Dj. and Dragovich (1997, 2000), G. Dj, Dragovich and Lj. Nesic (1999)].
- Adelic quantum mechanics: $(L_2(A), W(z), U(t))$
  - adelic Hilbert space, $L_2(A)$
  - Weyl quantization of complex-valued functions on adelic classical phase space, $W(z)$
  - unitary representation of an adelic evolution operator, $U(t)$
- The form of adelic wave function

$$\psi = \psi_\infty(x_\infty) \cdot \prod_{p \in M} \psi_p(x_p) \cdot \prod_{p \notin M} \Omega(|x|_p)$$

Adelic Quantum Theory

- **Exactly soluble p-adic and adelic quantum mechanical models:**
- a free particle and harmonic oscillator [VVZ, Dragovich]
- a particle in a constant field, [G. Dj, Dragovich]
- a free relativistic particle[G. Dj, Dragovich, Nesic]
- a harmonic oscillator with time-dependent frequency [G. Dj, Dragovich]
- **Resume of AQM:** AQM takes in account ordinary as well as p-adic effects and may me regarded as a starting point for construction of more complete quantum cosmology (and string theory …) . In the low energy limit AQM effectively becomes the ordinary one.

(ADELIC) QUANTUM COSMOLOGY

- **In the very beginning the Universe was in a quantum state, which should be described by a wave function (complex valued and depends on some real parameters).**
- **…**
- There is no Schroedinger and Wheeler-De Witt equation for cosmological models.
- Feynman's path integral method was exploited and minisuperspace cosmological models are investigated as a model of adelic quantum mechanics [Dragovich (1995), G Dj, Dragovich, Nesic and Volovich (2002), G.Dj and Nesic (2005, 2008, 2016)…].

(ADELIC) QUANTUM COSMOLOGY - minisuperspace

- Adelic minisuperspace quantum cosmology is an application of adelic quantum mechanics to the cosmological models.
- Path integral approach to standard quantum cosmology

$$< h_{ij}'', \phi'', \Sigma'' \mid h_{ij}', \phi', \Sigma' >_{\infty} = \int D(g_{\mu\nu})_{\infty} D(\phi)_{\infty} \chi_{\infty}(-S_{\infty}[g_{\mu\nu}, \phi])$$

$$< h_{ij}'', \phi'', \Sigma'' \mid h_{ij}', \phi', \Sigma' >_{p} = \int D(g_{\mu\nu})_{p} D(\phi)_{p} \chi_{p}(-S_{p}[g_{\mu\nu}, \phi])$$

Quantum Theory – path integrals

- The main problem in our approach is computation of *p*-adic transition amplitude in Feynman's PI method

$$K_p(x'',t'';x',t') = \int_{(x',t')}^{(x'',t'')} \chi_p\left(-\frac{1}{h}\int_{t'}^{t''} L(\dot{q},q,t)\right) Dq$$

- Exact general expression ($\bar{S}$-classical action)

$$K_p(x'',t'';x',t') = \lambda_p\left(-\frac{1}{2h}\frac{\partial^2 \bar{S}}{\partial x'\partial x''}\right) \times \left|\frac{1}{h}\frac{\partial^2 \bar{S}}{\partial x'\partial x''}\right|_p^{1/2} \chi_p\left(-\bar{S}(x'',t'';x',t')\right)$$

$$K_p(x'',y'',z'',t'';x',y',z',t') = \lambda_p\left(\det\begin{pmatrix} -\frac{1}{2}\frac{\partial^2\bar{S}}{\partial x'\partial x''} & -\frac{1}{2}\frac{\partial^2\bar{S}}{\partial x'\partial y''} & -\frac{1}{2}\frac{\partial^2\bar{S}}{\partial x'\partial z''} \\ -\frac{1}{2}\frac{\partial^2\bar{S}}{\partial y'\partial x''} & -\frac{1}{2}\frac{\partial^2\bar{S}}{\partial y'\partial y''} & -\frac{1}{2}\frac{\partial^2\bar{S}}{\partial y'\partial z''} \\ -\frac{1}{2}\frac{\partial^2\bar{S}}{\partial z'\partial x''} & -\frac{1}{2}\frac{\partial^2\bar{S}}{\partial z'\partial y''} & -\frac{1}{2}\frac{\partial^2\bar{S}}{\partial z'\partial z''} \end{pmatrix}\right)$$

$$\times \left|\det\begin{pmatrix} -\frac{\partial^2\bar{S}}{\partial x'\partial x''} & -\frac{\partial^2\bar{S}}{\partial x'\partial y''} & -\frac{\partial^2\bar{S}}{\partial x'\partial z''} \\ -\frac{\partial^2\bar{S}}{\partial y'\partial x''} & -\frac{\partial^2\bar{S}}{\partial y'\partial y''} & -\frac{\partial^2\bar{S}}{\partial y'\partial z''} \\ -\frac{\partial^2\bar{S}}{\partial z'\partial x''} & -\frac{\partial^2\bar{S}}{\partial z'\partial y''} & -\frac{\partial^2\bar{S}}{\partial z'\partial z''} \end{pmatrix}\right|_p^{1/2} \chi_p\left(-\bar{S}(x'',y'',z'',t'';x',y',z't')\right)$$

Tachyons

- A. Somerfeld - first discussed about possibility of particles to be faster than light (100 years ago).
- G. Feinberg - called them tachyons: Greek word, means fast, swift (almost 50 years ago).
- According to Special Relativity: $m^2 < 0, \quad v = \frac{p}{\sqrt{p^2 + m^2}}.$
- From a more modern perspective the idea of faster-than-light propagation is abandoned and the term "tachyon" is recycled to refer to a quantum field with $m^2 = V'' < 0.$

Tachyons

- Field Theory
- Standard Lagrangian (real scalar field):

$$L(\varphi, \partial_\mu \varphi) = T - V = \frac{1}{2}\partial_\mu \varphi \partial^\mu \varphi - V(\varphi_0) - V'(\varphi_0)\varphi - \frac{1}{2}V''(\varphi_0)\varphi^2 - ...$$

- Extremum (min or max of the potential): $V'(\varphi_0) = 0$
- Mass term: $V''(\varphi_0) = m^2$
- Clearly V'' can be negative (about a maximum of the potential). Fluctuations about such a point will be unstable: tachyons are associated with the presence of instability.

$$L(\phi, \partial_\mu \phi) = L_{kin} - V = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2}m^2\phi^2 + const$$

Tachyons-From Field Theory to Classical Analogue – DBI and Sen approach

- String Theory
- A. Sen – proposed (effective) tachyon field action (for the *Dp*-brane in string theory):

$$S = -\int d^{n+1}x V(T)\sqrt{1+\eta^{ij}\partial_i T\partial_j T}$$
$$\eta_{00} = -1$$
$$\eta_{\mu\nu} = \delta_{\mu\nu} \quad \mu,\nu = 1,\ldots,n$$

- $T(x)$ - tachyon field
- $V(T)$ - tachyon potential
- Non-standard Lagrangian and DBI Action!

Tachyons-From Field Theory to Classical Analogue – DBI and Sen approach

- Equation of motion (EoM):

$$\ddot{T}(t) - \frac{1}{V(T)}\frac{dV}{dT}\dot{T}^2(t) = -\frac{1}{V(T)}\frac{dV}{dT}$$

- Can we transform EoM of a class of non-standard Lagrangians in the form which corresponds to Lagrangian of a canonical form, even quadratic one? Some classical canonical transformation (CCT)?

Classical Canonical Transformation and Quantization

- CCT: $T, P \quad \mapsto \quad \tilde{T}, \tilde{P}$
- Generating function: $G(\tilde{T}, P) = -PF(\tilde{T})$

$$T = -\frac{\partial G}{\partial P} = F(\tilde{T}) \qquad \tilde{P} = -\frac{\partial G}{\partial \tilde{T}} \Rightarrow P = \left(\frac{dF(\tilde{T})}{d\tilde{T}}\right)^{-1} \tilde{P}$$

- EoM transforms to

$$\ddot{\tilde{T}} + \left(\frac{\dfrac{d^2 F(\tilde{T})}{d\tilde{T}^2}}{\dfrac{dF(\tilde{T})}{d\tilde{T}}} - \frac{dF(\tilde{T})}{d\tilde{T}} \frac{d \ln V(F)}{dF} \right) \dot{\tilde{T}}^2 + \frac{1}{\dfrac{F(\tilde{T})}{d\tilde{T}}} \frac{d \ln V(F)}{dF} = 0$$

Classical Canonical Transformation and Quantization

- Choice: $F^{-1}(T) = \int_{T_0}^{T} \frac{dX}{V(X)}$
- EoM reduces to:

$$\ddot{\tilde{T}} + \frac{1}{F'} \frac{d \ln V(F)}{dF} = 0!!!$$

- This EoM can be obtained from the standard type Lagrangians $\mathcal{L} = L_{kin} - V$

Classical Canonical Transformation and Quantization

- Example: $V(T)=\dfrac{1}{\cosh(\beta T)}$

$$F^{-1}(T)=\int^{T}\frac{dx}{V(x)}=\frac{1}{\beta}\sinh(\beta T)$$

- Generating function: $G(\tilde{T},P)=-PF(\tilde{T})=-\dfrac{P}{\beta}\operatorname{arcsinh}(\beta T)$
- EoM: $\ddot{\tilde{T}}(t)-\beta^2\tilde{T}(t)=0$
- This EoM can be obtained from the standard-type (quadratic) Lagrangian

$$\mathcal{L}_{quad}(\tilde{T},\dot{\tilde{T}})=\frac{1}{2}\dot{\tilde{T}}^2+\frac{1}{2}\beta^2\tilde{T}^2$$

Classical Canonical Transformation and Quantization

- Action (quadratic): $S_{cl} = \int_0^{\tau} \mathcal{L}_{quad} dt = \frac{\beta}{2}\left(\left(\tilde{T}_1^2 + \tilde{T}_2^2\right)\coth(\beta\tau) - \frac{2\tilde{T}_1\tilde{T}_2}{\sinh(\beta\tau)}\right)$
- Quantization: Transition (adelic!?) amplitude, $v = \infty, 2, 3, \ldots p, \ldots$

$$\mathcal{K}_v(\tilde{T}_2, \tau; \tilde{T}_1, 0) = \lambda_v\left(\frac{1}{2\tau}\right)\left|-\frac{1}{\tau}\right|_v^{1/2} \chi_v\left(-S_{cl}(\tilde{T}_2, \tau; \tilde{T}_1, 0)\right)$$

- The necessary condition for the existence of a *p*-adic (adelic) quantum model is the existence of a *p*-adic quantum-mechanical ground (vacuum) state in the form of a characteristic Ω -function; we get expression which defines constraints on parameters of the theory

$$\int_{|\tilde{T}_1|_p \leq 1} \mathcal{K}_p(\tilde{T}_2, \tau; \tilde{T}_1, 0) d\tilde{T}_1 = \Omega(|\tilde{T}_2|_p)$$

Classical Canonical Transformation and Quantization

- Using *p*-Adic Gauss integral

$$\int_{|y|_p\le 1}\chi_p(ay^2+by)dy=\begin{cases}\Omega(|b|_p), & |a|_p\le 1\\ \dfrac{\lambda_p(a)}{|a|_p^{1/2}}\chi_p(-\dfrac{b^2}{4a})\Omega(|\dfrac{b}{a}|_p), & |a|_p>1\end{cases}$$

- we get (in the case of an inverse power-law potential) $V\sim\tilde{T}^{-n}, n=1$

$$\lambda_p\left(\frac{1}{2\tau}\right)|\tau|_p^{-1/2}\ \chi_p\left(-\frac{1}{2\tau}\tilde{T}_2^2-\frac{1}{2}k\tau\tilde{T}_2+\frac{1}{24}k^2\tau^3\right)\times I_{Gauss}=\Omega(|\tilde{T}_2|_p)$$

Classical Canonical Transformation and Quantization

- Case 1 $|\tau|_p > 1$ impossible to fulfill
- Case 2 $|\tau|_p = 1$

$$\chi_p\left(-\frac{1}{2\tau}\tilde{T}_2^2 - \frac{1}{2}k\tau\tilde{T}_2 + \frac{1}{24}k^2\tau^3\right)\Omega(|\frac{\tilde{T}_2}{\tau} - \frac{1}{2}k\tau|_p) = \Omega(|\tilde{T}_2|_p)$$

- Case 3 $|\tau|_p < 1$

$$\chi_p\left(-k\tau\tilde{T}_2 + \frac{1}{6}k^2\tau^3\right)\Omega(|-2\tilde{T}_2 + k\tau^2|_p) = \Omega(|\tilde{T}_2|_p)$$

Quadratic and almost quadratic systems (*p*-adic case)

- We consider DBI-type Lagrangian in 4d
- $L_T = -V(T)\sqrt{1 + (\partial T)^2}$
- $ds_p^2 = -c^2 + a(t)^2(dx^2 + dy^2 + dz^2)$
- $a(t)$ – scale factor, T – tachyon field, $p = 1(mod\ 4)$
- Equation of motion
- $\ddot{T} + 3H(t)\dot{T}(1 - \dot{T}^2) + \frac{1}{V(T)}\frac{dV(T)}{dT}(1 - \dot{T}^2) = 0$
- Where, Hubble parameter is $H = \frac{\dot{a}}{a}$

The method of Darboux

- After performing necessary and straightforward integration

$$L_T \equiv a^3(t)\mathcal{L}_T = a^3(t)(-V(t)\sqrt{1-\dot{T}^2})$$

- **The method of Darboux**
- The problem of reconstructiong an adequate Lagrangian, starting from EoM – the inverse problem.
- The procedure of constructiong a Lagrangian is generaly simplied when:

$$\ddot{q} + A(q, \dot{q}, t) = 0$$

Equivalent, quadratic Lagrangian?

- $L = \int (\dot{q} - \omega)\Lambda(q\omega t)d\omega - \int A(\xi, \dot{q}_0, t)\Lambda(\xi, \dot{q}_0, t)d\xi + \frac{F(q,t)}{t}$
- Jacobi multiplayer $\Lambda(q, \dot{q}, t) = \chi_\nu \left(\int \frac{\partial A(q,\dot{q},t)}{\partial \dot{q}} dt \right)$
- Our initial equation now takes the form

$$\ddot{T} + 3H(t)(1 + \frac{2}{3}\frac{\dot{H}(t)}{H^2(t)})\dot{T} + \frac{V'(T)}{V(T)}(1 - \dot{T}^2) = 0$$

Equivalent, quadratic Lagrangian?

$$A(T,\dot{T},t) = 3H(t)\left(1+\frac{2}{3}\frac{\dot{H}(t)}{H^2(t)}\right)\dot{T} + \frac{V'(T)}{V(T)}\left(1-\dot{T}^2\right)$$

$$\Lambda(T,\dot{T},t) = \frac{a^3(t)H^2(t)}{V^2(T)}$$

- We are close...

$$L = a^3(t)H^2(t)\left[\frac{1}{2}\left(\frac{\dot{T}}{V(T)}\right)^2 + \frac{1}{2}\frac{1}{V^2(T)}\right]$$

$$\dot{\phi} = \frac{\dot{T}}{V(T)}$$

And some examples

$$\phi = \int^{T} \frac{dT}{V(T)}$$

$$L = a(t)\dot{a}^2(t)[\frac{1}{2}\dot{\phi}^2 + \frac{1}{2V^2(T(\phi))}]$$

- If we choose, for example, *exp* potential $V(T) = V_0 e^{-\omega T}, \quad V_0 = const,\ \omega = const$

- Inverted harmonic oscilator with time dependent mass

$$L = \frac{1}{2}m(t)\dot{x}^2 + \frac{1}{2}m(t)\omega^2 x^2$$

We get the final form

$$x(t) = \frac{\alpha}{\eta \sinh(\gamma'' - \gamma')} \left(\frac{\eta' x'}{\alpha'} \sinh(\gamma'' - \gamma) - \frac{\eta'' x''}{\alpha''} \sinh(\gamma' - \gamma) \right)$$

$$S_{cl}(x'', t''; x', t') = \frac{1}{2} m'' x''^2 \left(\frac{\dot{\alpha}''}{\alpha''} - \frac{\dot{\eta}''}{\eta''}\right) - \frac{1}{2} m' x'^2 \left(\frac{\dot{\alpha}'}{\alpha'} - \frac{\dot{\eta}'}{\eta'}\right) \coth(\gamma'' - \gamma') + \frac{1}{2}\left(m'' \dot{\gamma}'' x''^2 + m' \dot{\gamma}' x'^2\right) - \frac{\eta' \eta'' \sqrt{\dot{\gamma}' \dot{\gamma}''}\, x' x''}{\sinh(\gamma'' - \gamma')}$$

$$K(x'', t''; x', t') = F(t'', t') \chi_v(S_{cl}(x'', t''; x', t')/\mathrm{h})$$

$$F(t'', t') = \left[\frac{i}{2\pi\hbar} \frac{\partial^2}{\partial x' \partial x''} S_{cl}(x'', t''; x', t') \right]^{\frac{1}{2}}$$

Tachyon inflation

- Consider the tachyonic field *T* minimally coupled to Einstein's gravity

$$S = -\frac{1}{16\pi G}\int \sqrt{-g} R d^4x + S_T$$

- Where *R* is Ricci scalar, *g* – determinant of the metric tensor and tachyon action

$$S_T = \int \sqrt{-g}\mathcal{L}(T, \partial_\mu T) d^4x$$
$$\mathcal{L} = -V(T)\sqrt{1 + g^{\mu\nu}\partial_\mu T \partial_\nu T}$$

- Friedman equation:

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3M_{Pl}^2}\frac{V}{(1-\dot{T}^2)^{1/2}}$$

- Energy-momentum conservation equation:

$$\dot{\rho} = -3H(P + \rho)$$
$$\frac{\ddot{T}}{1-\dot{T}^2} + 3H\dot{T} + \frac{V'}{V} = 0$$

Energy density and pressure:

$$\rho = \frac{V(T)}{\sqrt{1-\dot{T}^2}}$$
$$P = -V(T)\sqrt{1-\dot{T}^2}$$

Tachyon inflation

- Introducing a constant dimensionless ratio

$$X_0 = \frac{\lambda T_0^2}{M_{Pl}^2}, \qquad \text{where } \lambda = \frac{M_s^4}{g_s (2\pi)^3}$$

$$\tau = t \cdot T_0$$

$$x = \frac{T}{T_0}$$

$$U(x) = \frac{V(x)}{\lambda}$$

$$\tilde{H} = \frac{H}{T_0}$$

- The system of dimensionless equation is obtained

$$\tilde{H}^2 = \frac{X_0^2}{3} \frac{U(x)}{\sqrt{1-\dot{x}^2}}$$

$$\ddot{x} + X_0 \sqrt{3U(x)(1-\dot{x}^2)^{3/2}}\,\dot{x} + \frac{(1-\dot{x}^2)}{U(x)} \frac{dU(x)}{dx} = 0$$

- In addition, the Friedman acceleration equation

$$\dot{\tilde{H}} = -\frac{X_0^2}{2}(\tilde{P} + \tilde{\rho})$$

$$\tilde{\rho} = \frac{U(T)}{\sqrt{1-\dot{x}^2}}$$

$$\tilde{P} = -U(x)\sqrt{1-\dot{x}^2}$$

Tachyon inflation

- The slow-roll parameters

$$\epsilon_{i+1} \equiv \frac{d \ln |\epsilon_i|}{dN}, \quad i \geq 0, \quad \epsilon_0 \equiv \frac{H_*}{H}$$

$$\epsilon_1 = -\frac{\dot{H}}{H^2}, \quad \epsilon_2 = \frac{1}{H}\frac{\ddot{H}}{\dot{H}} + 2\epsilon_1$$

$$\epsilon_1 = \frac{3}{2}\dot{T}^2, \quad \epsilon_2 = 2\frac{\ddot{T}}{H\dot{T}}$$

- Number of e-folds

$$N(t) = \int_{t_i}^{t_e} H(t)dt$$

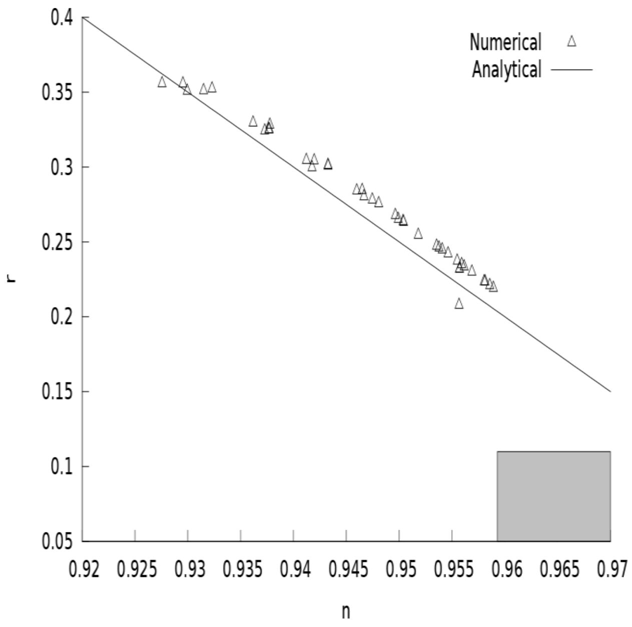
- In the slow-roll aproximation

$$N(x) = X_0^2 \int_{x_i}^{x_e} \frac{U(x)^2}{|U'(x)|} dx, \qquad \text{where } \varepsilon_1(x_e) = 1$$

- Observational parameters
  - The scalar spectral index $n = 1 - 2\epsilon_1(x_i) - \epsilon_2(x_i)$
  - The tensor-to-scalar ratio $r = 16\epsilon_1(x_i)$

Tachyon inflation

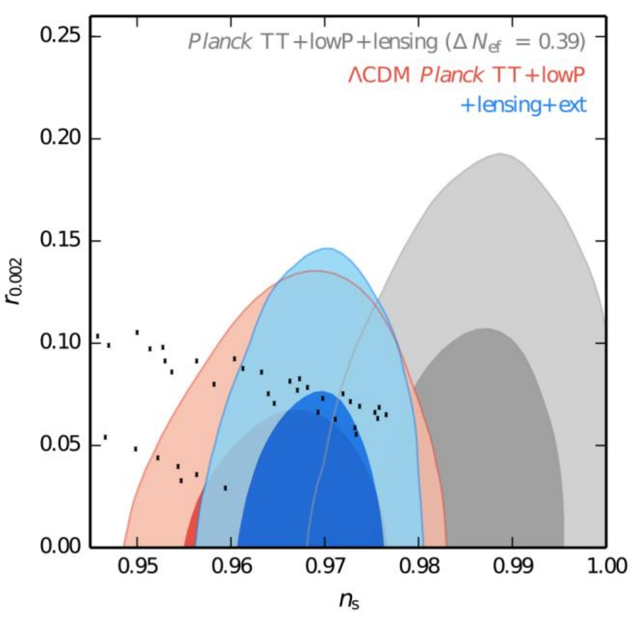
- Numerical results



$$45 \leq N \leq 75,\ 5 \leq X_0 \leq 25$$

$$U(x) = \frac{1}{\tilde{x}^4} \qquad \text{(left)}$$

$$U(x) = \frac{1}{\cosh(\tilde{x})} \qquad \text{(right)}$$



Randall Sundrum Model

- Braneworld cosmology, RSII metric

$$ds_{(5)}^2 = (e^{-2ky} + \phi) g^{\mu\nu} dx^\mu dx^\nu - \left(\frac{e^{-2ky}}{e^{-2ky} + \phi} \right)^2 dy^2$$

- Radion field: $\phi(x)$
- Consider an additional 3-brane moving in the bulk; The 5th coordinate *y(x)* can be treated as a dynamical scalar (tachyon) field $\theta(x) = k^{-1} e^{ky(x)}$

$$S_{\text{brane}} = -\int d^4x \sqrt{-g} \frac{\sigma}{k^4\theta^4} (1 + k^2\theta^2\phi)^2 \sqrt{1 - \frac{g^{\mu\nu}\theta_{,\mu}\theta_{,\nu}}{(1 + k^2\theta^2\phi)^3}}$$

$$\phi = 0 \quad \Rightarrow \quad S_{\text{brane}}^{(0)} = -\int d^4x \sqrt{-g} \frac{\sigma}{k^4\theta^4} \sqrt{1 - g^{\mu\nu}\theta_{,\mu}\theta_{,\nu}}$$

Randal-Sundrum model and tachyon-like inflation

- Inflation is driven by the tachyon field originating in string theory
- A simple model of this kind is based on the second Randall-Sundrum (RSII) model
- The RSII model is a 4+1 dimensional Anti de Sitter (AdS_5) universe containing two 3-branes with opposite tensions separated in the fifth dimension: observers reside on the positive tension brane and the negative tension brane is pushed off to infinity
- The fluctuation of the interbrane distance along the extra dimension implies the existence of the radion.
- Radion - a massless scalar field that causes a distortion of the bulk geometry.
- The bulk spacetime of the extended RSII model in Fefferman-Graham coordinates is described by the metric

The spatially flat FRW metric

$$ds_{(5)}^2 = G_{ab} dX^a dX^b = \frac{1}{k^2 z^2}\left[\left(1 + k^2 z^2 \eta(x)\right) g^{\mu\nu} dx^\mu dx^\nu - \frac{1}{\left(1 + k^2 z^2 \eta(x)\right)^2} dz^2\right]$$

The bulk space-time metric

Randal-Sundrum model and tachyon-like inflation

- The brane Lagrangian, after integrating out the fifth coordinate

$$S=\int d^4x\sqrt{-g}\left(-\frac{R}{16\pi G}+\frac{1}{2}g^{\mu\nu}\Phi_{,\mu}\Phi_{,\nu}\right)\underbrace{-\int d^4x\sqrt{-g}\,\frac{\sigma}{k^4\Theta^4}(1+k^2\Theta^2\eta)^2\sqrt{1-\frac{g^{\mu\nu}\Theta_{,\mu}\Theta_{,\nu}}{(1+k^2\Theta^2\eta)^3}}}_{S_{br}}$$

- Where *Φ* is the radion field, *Θ* is the tachyon field, *k* is the inverse of the AdS curvature radius $k=1/\ell$ and η is $\eta=\sinh^2\left(\sqrt{4/3\pi G}\Phi\right)$
- In the absence of radion S_{br} is the tachyon condensate $S_{\rm br}^{(0)}=-\int d^4x\sqrt{-g}\,\frac{\lambda}{\Theta^4}\sqrt{1-g^{\mu\nu}\Theta_{,\mu}\Theta_{,\nu}}$
- The combined brane-radion Lagrangian is

$$\mathcal{L}=\frac{1}{2}g^{\mu\nu}\Phi_{,\mu}\Phi_{,\nu}-\frac{\lambda\psi^2}{\Theta^4}\sqrt{1-\frac{g^{\mu\nu}\Theta_{,\mu}\Theta_{,\nu}}{\psi^3}}$$

the brane tension

$$\lambda=\frac{\sigma}{k^4}\qquad \psi=1+k^2\Theta^2\eta$$

- In the spatially flat RS cosmology the Hubble expansion rate *H* is related to Hamiltonian via modified Friedman equation

$$H\equiv\frac{\dot{a}}{a}=\sqrt{\frac{8\pi G}{3}\mathcal{H}\left(1+\frac{2\pi G}{3k^2}\mathcal{H}\right)}$$

Randal-Sundrum model

- The dimensionless Hamiltonian's equations are obtained

$$\dot{\phi} = \pi_\phi$$

$$\dot{\theta} = \frac{\theta^4 \psi \pi_\theta}{\sqrt{1+\theta^8 \pi_\theta^2 / \psi}}$$

$$\dot{\pi}_\phi = -3h\pi_\phi - \frac{\psi}{2\theta^2}\frac{4+3\theta^8\pi_\theta^2/\psi}{\sqrt{1+\theta^8\pi_\theta^2/\psi}}\eta'$$

$$\dot{\pi}_\theta = -3h\pi_\theta + \frac{\psi}{\theta^5}\frac{4-3\theta^{10}\eta\pi_\theta^2/\psi}{\sqrt{1+\theta^8\pi_\theta^2/\psi}}$$

$$\dot{h} = -\frac{\kappa^2}{2}(\bar{\rho}+\bar{p})\left(1+\frac{\kappa^2}{6}\bar{\rho}\right)$$

$$\dot{N} = h$$

Additional equations, solved in parallel

A combined dimensionless coupling →

$$\kappa^2 = 8\pi\lambda G k^2$$

The Hubble expansion rate →

$$h \equiv \frac{\dot{a}}{a} = \sqrt{\frac{\kappa^2}{3}\bar{\rho}\left(1+\frac{\kappa^2}{12}\bar{\rho}\right)}$$

$$\psi = 1+\theta^2\eta,$$

$$\eta = \sinh^2\left(\sqrt{\frac{\kappa^2}{6}}\phi\right),$$

$$\eta' = \frac{d\eta}{d\phi} = \sqrt{\frac{\kappa^2}{6}}\sinh\left(\sqrt{\frac{2\kappa^2}{3}}\phi\right),$$

preassure →

$$\bar{p} = \frac{1}{2}\dot{\phi}^2 - \frac{\psi^2}{\theta^4}\sqrt{1-\dot{\theta}^2/\psi^3},$$

energy density →

$$\bar{\rho} = \frac{1}{2}\dot{\phi}^2 + \frac{\psi^2}{\theta^4}\frac{1}{\sqrt{1-\dot{\theta}^2/\psi^3}}$$

Randal-Sundrum model

- Slow-roll parameters are

$$\epsilon_0 \equiv \frac{H_*}{H} \qquad H_*\text{-Hubble rate at an arbitrarily chosen time}$$

$$\epsilon_i \equiv \frac{d \ln |\epsilon_{i-1}|}{H dt}, \qquad i \geq 1$$

- Observational parameters: the tensor-to-scalar ration (*r*) and scalar spectral index (n_s)

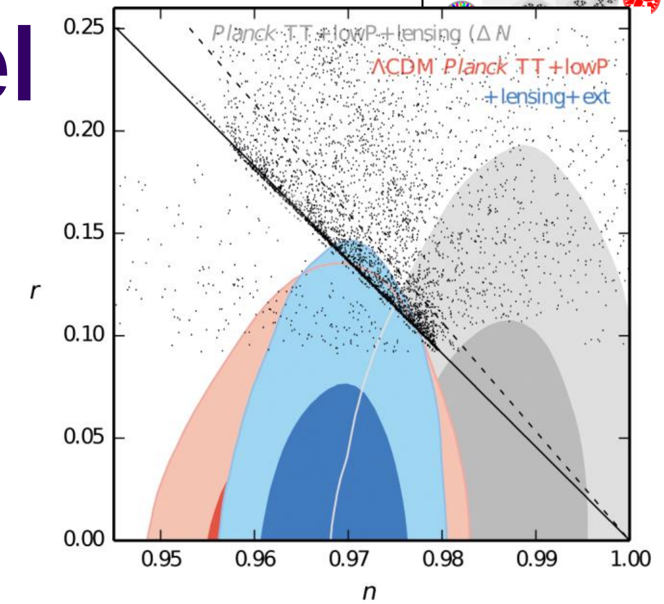
$$r = 16\epsilon_1(\theta_i)\left[1 - \frac{1}{6}\epsilon_1(\theta_i) + C\epsilon_2(\theta_i)\right]$$

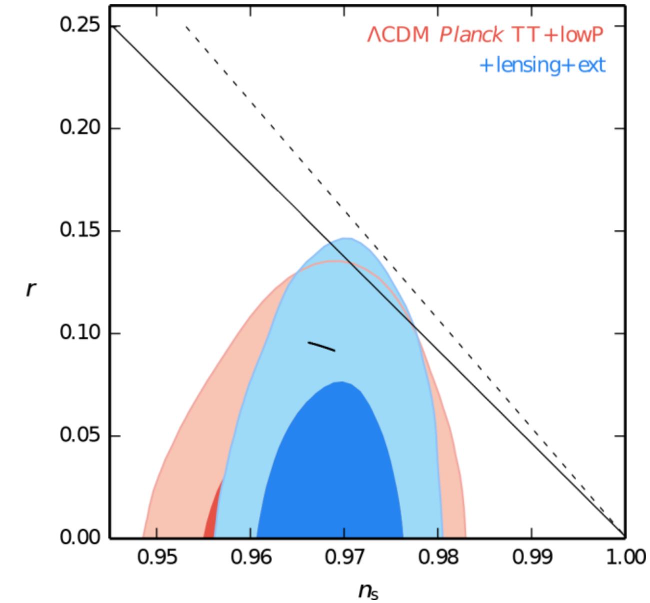
$$n_s = 1 - 2\epsilon_1(\theta_i) - \epsilon_2(\theta_i) - \left[2\epsilon_1^2(\theta_i) + \left(2C + \frac{8}{3}\right)\epsilon_1(\theta_i)\epsilon_2(\theta_i) + C\epsilon_2(\theta_i)\epsilon_3(\theta_i)\right]$$

- Numerical results:

$$60 \leq N \leq 120,\ 1 \leq \kappa \leq 12 \text{ and } 0 \leq \phi_0 \leq 0.5 \text{ (top)}$$

$$115 \leq N \leq 120,\ \phi_0 = 0.05,\ \kappa = 1.25 \text{ (bottom)}$$





Beyond

- Our understanding of tachyon matter, especially its quantum aspects is still quite pure.
- Perturbative solutions for classical particles analogous to the tachyons offer many possibilities in quantum mechanics, quantum and string field theory and cosmology on archimedean and nonarchimedean spaces.
- Reverse Engineering Method-REM remains a valuable auxiliary tool for investigation on tachyonic–universe evolution for nontrivial models.
- It was shown that the theory of *p*-adic inflation can be compatible with CMB observations. Quantization of tachyons could allow us to consider even more realistic inflationary models including quantum fluctuations and to test their p-adic aspects.
- Attractor behavior of the original DBI based Lagrangian model has been approved in real case, but not yet tested in a *p*-adic and adelic context

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Thank you!